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The VIX-Derived Volatility Model: A VIX-first Joint SPX-VIX Framework

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Summary:

The CBOE volatility index (VIX) measures the expected future volatility of the S&P 500 (SPX), derived from the SPX option market. The VIX is actively traded in a futures market, and there is also a market for European options on the VIX.

Market participants use SPX options to hedge VIX exposure and vice versa, making it crucial to have a consistent model that jointly describes the dynamics of the SPX and VIX derivatives markets. However, according to Gatheral et al. (2020), "Designing a model that jointly calibrates SPX and VIX options prices is known to be extremely challenging. Indeed, this problem is sometimes considered to be the holy grail of volatility modeling. We will simply refer to it as the joint calibration problem."

In recent years, several contributions have been made to address this SPX/VIX joint calibration problem. One initial approach proposes stochastic volatility models to solve the problem. Specifically, these models consider the instantaneous variance of the SPX, v_t, as a Markov semimartingale, from which the dynamics of the SPX and VIX are obtained. Among these models, we can cite Gatheral (2008), who proposes a CEV model, Baldeaux and Badran (2014), who work with a 3/2 model with jumps, and Fouque and Saporito (2018) who use a generalization of the Heston model. According to the authors of the paper we are analyzing, this type of model is only "partially successful at fitting to the market."

Alternative approaches have been developed, such as *Gaussian polynomial volatility models* (Abi-Jaber *et al*. (2025)), *signature models* (Cuchiero *et al*. (2025)), or the use of martingale optimal transport methods (Guo *et al.* (2022), Guyon (2024)). According to the authors, these alternative models produce "promising numerical results," but at the cost of increased model complexity, leading to "long calibration times, unstable parameters, and results that are difficult to interpret in practice."

The paper proposes a new model called the *VIX-derived volatility* (VDV) framework. The VDV is a stochastic volatility model in which the instantaneous variance of the SPX, v_t, is considered an adapted stochastic process. The difference between the VDV model and classical stochastic volatility models lies in the fact that the authors do not introduce a specific dynamic for v_t from the outset. Instead, this process is determined by the dynamics of the VIX process, X_t, which is explicitly included in the initial model specification. The instantaneous variance is obtained through a smooth coupling map such that $v_t = \psi_t(X_t)$. The specific form of this map depends on the VIX process and can be explicitly defined for some specific cases, such as Geometric Brownian Motion and polynomial processes. For the remaining cases, the authors develop a numerical approach.

The general VDV model is given by:

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \sqrt{v_t}\,dW_t^{(S)} \\
v_t &= \psi_t(X_t) \\
dX_t &= \mu(t, X_t)dt + \sigma(t, X_t)dW_t^{(X)}
\end{aligned}
$$

where S_t is the discounted SPX process, $\mu(t, X_t)$ and $\sigma(t, X_t)$ are, respectively, the drift and diffusion terms of the Itô process determining the dynamics of the VIX process, X_t, and $W_t^{(S)}$, $W_t^{(X)}$ are two correlated Brownian motions with respect to the equivalent martingale measure $\mathbb{Q}$, with correlation ρ.

The crucial element of the model is the function ψ_t, called the *coupling function*, which relates the VIX to the instantaneous variance. For the model to be well-defined, the coupling function must be positive and measurable, so that v_t is adapted and almost surely positive. Furthermore, it must satisfy the so-called *consistency condition*; that is, the VIX value obtained from v_t through its definition must match the specification of X_t in the model.

The authors study different model specifications. The first, called *VIX as Geometric Brownian Motion* (GBM), consists of setting $\mu(t, X_t) = 0$ and $\sigma(t, X_t) = \sigma X_t$. For this model, the authors obtain the explicit form of the coupling function and prove its consistency. They also apply the model to the valuation of options on the SPX and the VIX using Monte-Carlo simulation, obtaining a flat implied volatility for the VIX (since it is a GBM) and flexibility around the skew of the SPX by varying the correlation parameter.

The authors generalize the result of the GBM to polynomial processes, finding consistency if the process is time-homogeneous. Moreover, they derive an approximate consistent coupling function when X_t is a time-inhomogeneous non-polynomial process.

In order to assess the numerical accuracy of the approximate method, they consider the instantaneous volatility of the Heston model, and obtain from it the coupling function explicitly as a second-degree polynomial. Then they apply the approximate method to recover the same function, finding polynomial coefficients very close to the exact ones.

The last specification of the VDV model, called the *Local-Volatility-VIX* model, specifies a mean-reverting local volatility process for the VIX, determined by $\mu(t, X_t) = k(\theta_t - X_t)$ and $\sigma(t, X_t) = \sigma_{LV}(t, X_t)X_t$, where k is the mean-reversion speed, and θ_t is the long-term mean. A key feature of this model is that every parameter is used to calibrate at most one type of market data (θ_t for VIX futures, σ_{LV} for VIX options, and k, ρ for SPX options), making the parameter impact clear and simple.

The authors apply the Local-Volatility-VIX model to real data of the SPX and VIX obtaining the following achievements:

- The VIX process is flexible enough to be calibrated to VIX futures and VIX options.
- The derived instantaneous variance provides consistency between the SPX and the VIX.

- By setting the parameter ρ, the model can be calibrated to the SPX option market, providing a good model fit across all maturities.

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