$$\sigma_{-}^{2}(X) = \mathbb{E}\left\{\left[(X - \mathbb{E}(X))^{-}\right]^{2}\right\}$$

Semivariance:

Shortly after publishing the paper outlining the Mean-Variance (MV) analysis (Markowitz (1952)), which earned him the Nobel Prize in Economics, Harry Markowitz realized that variance was not very appropriate as a measure of risk, since it "considers extremely high and extremely low returns equally undesirable" (Markowitz (1959)). This led him to include semivariance, a downside risk measure whose expression appears above, into the MV analysis. The resulting Mean-Semivariance portfolio optimization problem was embedded in his Critical Line Algorithm (Markowitz et al. (1993)).

More recently, Ballestero (2005) proposed a stochastic programming model in which the portfolio semivariance is the objective function to be minimized subject to standard parametric constraints, giving rise to the Mean-Semivariance efficient frontier.

In the formula:

- X is a random variable interpreted as the future net worth of a portfolio at a given horizon.
- Y^{-} is equal to Y if Y is negative and equal to zero if Y is positive or zero.

From the current perspective, neither variance nor semivariance is considered an adequate risk measure because they are not coherent measures (variance does not satisfy the positive homogeneity property, and semivariance does not satisfy translation invariance) (see, for example, Noguer et al. (2025), Chapter 5).

References:

- Ballestero (2005): "Mean-Semivariance Efficient Frontier: A Downside Risk Model for Portfolio Selection", Applied Mathematical Finance, 12 (1), 1-15.
- Markowitz (1952): "Portfolio Selection", Journal of Finance, 7 (1), 77-91.
- Markowitz (1959): "Portfolio Selection: Efficient Diversification of Investments", Yale University Press.
- Markowitz, Tod, Xu and Yamane (1993): "Computation of Mean-Semivariance Efficient Sets by the Critical Line Algorithm", Annals of Operations Research, 45, 307-317.
- Noguer, Antolín and Bueno-Guerrero (2025): "Quantitative Portfolio Optimization: Advanced Techniques and Applications", Wiley.